32.11 An office building seeks to reduce heat losses through the windows by 75% by installing insulating shades. The design conditions for winter are $10°F$ outside, $68°F$ inside. The existing windows are $\frac{1}{8}in$ thick single pane glass with thermal conductivity $k = 0.7\frac{Btu}{hr \cdot ft \cdot °F}$. There will be an air gap between the windows and the shades, providing additional resistance of $R = 1.2\frac{hr \cdot ft^2 \cdot °F}{Btu}$. The shades have a thermal conductivity of $k = 0.05\frac{Btu}{hr \cdot ft \cdot °F}$. The convective heat transfer coefficients inside and outside are $1.5\frac{Btu}{hr \cdot ft^2 \cdot °F}$ and $4\frac{Btu}{hr \cdot ft^2 \cdot °F}$, respectively. What is the minimum thickness of the shades required to achieve the desired reduction in energy loss?

A. $\frac{1}{4}in$

B. $1in$

C. $2in$

D. $3in$

Treat the window as a Composite Wall. Consider the original arrangement without shades as State 1, and consider the updated arrangement with shades as State 2.

The heat transfer through the wall is given by $\dot{Q} = UA\Delta T$ which may be written on a per unit area basis as $\dot{q} = \frac{\dot{Q}}{A} = U\Delta T$. The overall coefficient of heat transfer, U, is the reciprocal of the total thermal resistance, R. Therefore, the heat flux can be expressed as $\dot{q} = \frac{\Delta T}{R}$. If the heat flux is reduced by 75%, then the ratio of the new heat flux (with shades) to the original heat flux (without shades) can be written and used to relate the total resistances for States 1 and 2. Note the temperature differential is the same and therefore cancels out. As a sense check, four times the thermal resistance reduces the heat transfer to a quarter of its original magnitude.

$$\frac{\dot{q}_2}{\dot{q}_1} = 0.25 = \frac{\frac{\Delta T}{R_{T,2}}}{\frac{\Delta T}{R_{T,1}}} = \frac{R_{T,1}}{R_{T,2}}$$

$$R_{T,2} = (4)\, R_{T,1}$$

Calculate the total thermal resistance for State 1. The resistance for the glass depends on thickness and thermal conductivity.

$$R_{T,1} = \frac{1}{h_i} + \frac{L_{glass}}{k_{glass}} + \frac{1}{h_o} = \frac{1}{1.5\frac{Btu}{hr \cdot ft^2 \cdot °F}} + \frac{\left(\frac{1}{8}in\right)\left(\frac{1ft}{12in}\right)}{0.7\frac{Btu}{hr \cdot ft \cdot °F}} + \frac{1}{4\frac{Btu}{hr \cdot ft^2 \cdot °F}} = 0.93\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

Determine the total thermal resistance for State 2.

$$R_{T,2} = (4)\, R_{T,1} = (4)\left(0.93\frac{hr \cdot ft^2 \cdot °F}{Btu}\right) = 3.73\frac{hr \cdot ft^2 \cdot °F}{Btu}$$

Write an expression for the thermal resistance at State 2, which has 2 additional terms for the shades and the air gap.

$$R_{T,2} = \frac{1}{h_i} + \frac{L_{shades}}{k_{shades}} + R_{air\,gap} + \frac{L_{glass}}{k_{glass}} + \frac{1}{h_o}$$

Since the magnitude of the 3 terms from the resistance for State 1 are known, substitute the numerical value and generate an expression where the thickness of the shades is the only unknown.

$$R_{T,2} = \frac{L_{shades}}{k_{shades}} + R_{air\,gap} + \left(\frac{1}{h_i} + \frac{L_{glass}}{k_{glass}} + \frac{1}{h_o}\right)$$

$$R_{T,2} = \frac{L_{shades}}{k_{shades}} + R_{air\,gap} + R_{T,1}$$

$$R_{T,2} = \frac{L_{shades}}{k_{shades}} + R_{air\,gap} + 0.93\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} = 3.73\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

$$\frac{L_{shades}}{k_{shades}} + R_{air\,gap} = 2.80\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

Substitute the thermal conductivity of the shades and the resistance of the air gap and solve for the shade thickness. Convert to inches.

$$\frac{L_{shades}}{0.05\frac{Btu}{hr \cdot ft \cdot {}^\circ F}} + 1.2\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu} = 2.8\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

$$\frac{L_{shades}}{0.05\frac{Btu}{hr \cdot ft \cdot {}^\circ F}} = 1.6\frac{hr \cdot ft^2 \cdot {}^\circ F}{Btu}$$

$$L_{shades} = 0.08ft\left(\frac{12in}{ft}\right) = 0.96in$$

Answer B